Final Examination - 2024

Mathematics (Standard)

(For all categories of candidates)

Subject code - 1306

Time: 3 hours 15 minutes Total marks: 80

The figures in the margin indicates full marks

I. Choose the correct option : $1 \times 10 = 10$

1. The product of two irrational numbers is
   (a) irrational number (b) rational number
   (c) may be rational or irrational (d) none of these

$\sqrt{3} \times \sqrt{3} = 3$

$\sqrt{3} \times \sqrt{9}$

$= \sqrt{27}$

2. $3, 8, 13, 18, \ldots$ is 78

a_n

first term

$a_n = a_1 + (n-1)d$

$78 = 3 + (n-1) \times (8-3)$

$78 = 3 + (n-1)5$

$78 - 3 = (n-1)5$

$75 = (n-1) \times 5$

$\frac{75}{5} = (n-1) \times \frac{5}{5}$

$15 = (n-1)$

$(n-1) = 15$

$n = 15 + 1$

$\boxed{n = 16}$ (c)

3. $f(x) = \underline{5x^2 + 13x + k}$

$f(x) = 0, \quad \underbrace{x_1, x_2}$

$$\boxed{\frac{ax^2 + bx + c}{x}} = 0$$

$$\frac{c}{a}$$

$$\frac{f(x)}{5} \qquad \frac{5x^2}{5} + \frac{13x}{5} + \frac{k}{5} = 0$$

$$x_1 \times \frac{1}{x_1} = \frac{k}{5}$$

$$\frac{k}{5} = 1, \quad \boxed{k = 5} \text{ (d)}$$

4. $x^2 - 2x - 8 = 0$

$x^2 - 2x - 4 - 4 = 0$

$\underbrace{x^2 - 4}_{} - 2x - 4 = 0$

$a^2 - b^2 = (a+b)(a-b)$

$\underbrace{x^2 - 2^2}_{} - \underbrace{2x - 4}_{} = 0$

$(x-2)\underbrace{(x+2)}_{} - 2\underbrace{(x+2)}_{} = 0$

$(x+2)(x-2-2) = 0$

$(x+2)(x-4) = 0$

$x + 2 = 0 \quad$ or $\quad x - 4 = 0$

$x = -2, \qquad x = 4$

$(-2, 4), \quad (4, -2)$ (a)

$x = -2$

$(-2)^2 - 2(-2) - 8 = 0$

$4 + 4 - 8 = 0$ ✓

$x = 4.$

$\underbrace{4^2}_{} - 2 \times 4 - 8 =$

$16 - 8 - 8 = 0$ ✓

5. (k)

$$\boxed{3x+2y=5.} \qquad \boxed{6x+ky=10}$$

l_1, l_2 $\qquad$ l_2

3, 2. $\qquad$ 6, k

$\boxed{k=4}$ (a)

$3x+2y=5$

$$\frac{6x}{2}+\frac{4y}{2}=\frac{10}{2}$$

$$3x+2y=5$$

6. $P(2,-2)$ and $Q(-1,y)$ $\qquad$ (x_1,y_1) (x_2,y_2)

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$5=\sqrt{(-1-2)^2+[y-(-2)]^2}$$

$$5=\sqrt{(-3)^2+(y+2)^2}$$

$$5=\sqrt{9+(y+2)^2}$$

Squaring both sides of the equation

$$5^2 = 9+(y+2)^2$$

$$9+(y+2)^2=25$$

$$(y+2)^2=25-9$$

$(y+2)^2=16,$ $\qquad$ $-2,-6$

$(-2+2)^2=0,\ \neq 16$ $\qquad$ ~~(a)~~

$2, -6$ ✓

$(2+2)^2 = 16,$

$(-6+2)^2 = (-4)^2 = 16$ (b) ✓

$y \; \underline{(2,-6)}$

7. If $\sin(2\theta) = 1,$ $\quad \underline{\tan\theta = 1}$

$\sin(\theta+\theta) = 1,$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin\theta\cos\theta + \cos\theta\sin\theta = 1$

$2\dfrac{\sin\theta\cos\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$ $\qquad \tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$2 \cdot \left(\dfrac{\sin\theta}{\cos\theta}\right) = \sec^2\theta$

$2 \cdot \tan\theta = \sec^2\theta \quad | \quad 1+\tan^2\theta = \sec^2\theta$

$\underline{2\tan\theta} = \underline{1+\tan^2\theta}$

$\tan^2\theta - 2\tan\theta + 1 = 0 \quad | \quad a^2 - 2 \cdot a \cdot b + b^2$

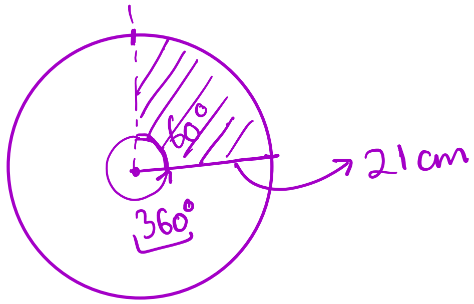
$(a-b)^2$

$(\tan\theta - 1)^2 = 0$

$\tan\theta - 1 = 0$

$\boxed{\tan\theta = 1}$ (c)

8.



$r = 21$ cm

Let's find the area $= \pi r^2$

$= \pi \times (21)^2$ cm^2

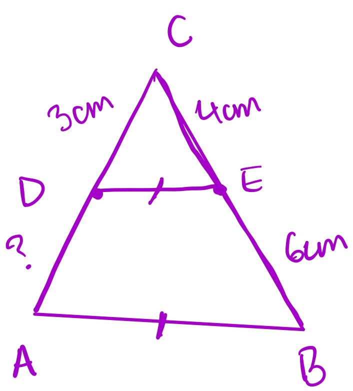
The required area $= \dfrac{60^\circ}{360^\circ} \times \pi \times (21)^2$

$= \dfrac{1}{\cancel{6}_{\cancel{2}}} \times \dfrac{\cancel{22}^{11}}{\cancel{7}} \times \cancel{21}^{3} \times 21$ cm^2

$= 231$ cm^2

(C)

9.



$DE \parallel AB$

$\dfrac{CD}{DA} = \dfrac{CE}{EB}$

[If a line is drawn parallel to one side of a triangle to intersect the other two sides at two distinct points, then the other two sides are divided in the same ratio]

$\dfrac{3}{DA} = \dfrac{4}{6}$

$DA = \dfrac{6 \times 3}{4} = \dfrac{18}{4} = \dfrac{9}{2} = 4.5$ cm.

(C)

10. Three coins are tossed.

$C_1 \quad C_2 \quad C_3$

H

T

$2 \times 2 \times 2$

$= 8$ possible outcomes

✓HHH ~~TTT~~

✓HHT ~~HTT~~

✓HTH ~~THT~~

✓THH ~~TTH~~

$= \frac{4}{8} = \frac{1}{2}$ (d)

II.

11. 96 404

$$\begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$$\begin{array}{r|l} 2 & 404 \\ \hline 2 & 202 \\ \hline & 101 \end{array}$$

$404 = 2 \times 2 \times 101$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$HCF(96, 404) = 2 \times 2 = 4.$

12. $2, 5, 8, \ldots$ $\to a_1$

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \cdot 2 + (10-1) \times 3]$

$d = 8 - 5$
$= 3$

$S_{10} = 5[4 + 9 \times 3] = 5 \times (4 + 27)$
$= 5 \times 31 = 155.$

13. $P(-5, 7)$, $Q(-1, 3)$

(x_1, y_1) , (x_2, y_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-1-(-5))^2 + (3-7)^2}$$

$$= \sqrt{(-1+5)^2 + (-4)^2} = \sqrt{4^2 + (-4)^2}$$

$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$= \sqrt{2 \times 16}$$

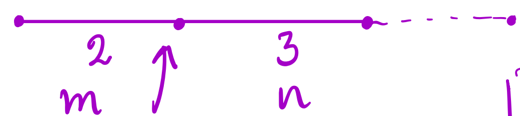
$$= \sqrt{2} \times \sqrt{16}$$

$$= \boxed{4\sqrt{2}}$$

$\sqrt{\ } \to$ -ve answer.

14. (x_1, y_1) (x_2, y_2)

$P(-1, 7)$ $M(x, y)$ $Q(4, -3)$



$$\boxed{\frac{m}{n} = \frac{2}{3}} = a$$

$$M\ (x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \qquad m = \frac{2n}{3}$$

$$= \left(\frac{\frac{2n}{3} \times 4 + n \times (-1)}{\frac{2n}{3} + n}, \frac{\frac{2n}{3} \times (-3) + n \times 7}{\frac{2n}{3} + n} \right)$$

$$\equiv \left(\frac{\frac{8n}{3} - n}{\frac{2n}{3} + n}, \frac{-\frac{6n}{3} + 7n}{\frac{2n}{3} + n} \right)$$

$$\equiv \left(\frac{\cancel{n}\left(\frac{8}{3} - 1\right)}{\cancel{n}\left(\frac{2}{3} + 1\right)}, \frac{\cancel{n}\left(-2 + 7\right)}{\cancel{n}\left(\frac{2}{3} + 1\right)} \right)$$

$$\equiv \left(\frac{\frac{8-3}{3}}{\frac{2+3}{3}}, \frac{5}{\frac{2+3}{3}} \right) \equiv \left(\frac{\frac{\cancel{5}}{\cancel{3}}}{\frac{\cancel{5}}{\cancel{3}}}, \frac{5 \times 3}{\cancel{5}} \right)$$

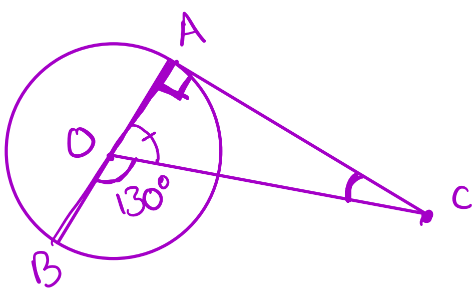
$$M(x,y) \equiv (1, 3)$$

15. $x \tan 45^\circ \sin 30^\circ = \cos 30^\circ \tan 30^\circ$

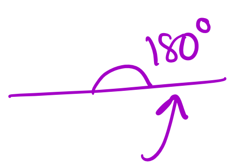
$$x \times 1 \times \frac{1}{2} = \frac{\cancel{\sqrt{3}}}{2} \times \frac{1}{\cancel{\sqrt{3}}}$$

$$\frac{x}{2} = \frac{1}{2} \quad \Big| \quad \frac{x}{\cancel{2}} \times \cancel{2} = \frac{1}{\cancel{2}} \times \cancel{2}$$

$$\boxed{x = 1}$$

16. 

$\angle ACO = ?$



$\angle AOC = ? = 180° - 130° = 50°$

ΔAOC, using angle sum property

$$\angle ACO + \angle AOC + \angle OAC = 180°$$

$$\angle ACO + 50° + 90° = 180°$$

[tangent is perpendicular to the radius at the point of tangency]

$$\angle ACO = 180° - 140°$$
$$= \boxed{40°}$$

17. $$\frac{1+\tan^2\theta}{1+\cot^2\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$$

$$\frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\frac{\cancel{\cos^2\theta+\sin^2\theta}\ 1}{\cos^2\theta}}{\frac{\cancel{\sin^2\theta+\cos^2\theta}\ 1}{\sin^2\theta}}$$

$[\cos^2\theta + \sin^2\theta = 1]$

$$= \frac{\sin^2\theta}{\cos^2\theta} = \boxed{\tan^2\theta}$$

18. $ax^2 + bx + c = 0$

Sum of zeros $= \boxed{\frac{-b}{a} = 4}$

product of zeros $= \boxed{\frac{c}{a} = -6}$

$\frac{-b}{a} = 4$ $\qquad\qquad$ $\frac{c}{a} = -6$

$b = -4a,$ $\qquad\qquad$ $c = -6a$

$ax^2 + bx + c = 0$

$ax^2 + (-4a)x + (-6a) = 0$

$ax^2 - 4ax - 6a = 0$ $\qquad$ [Divide by a]

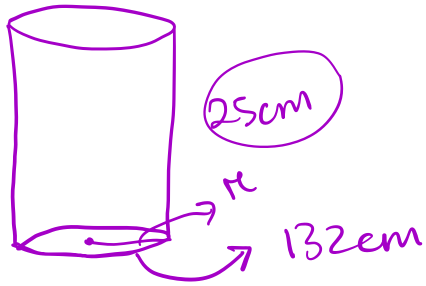
$\frac{1}{\cancel{a}}\left[\cancel{a}x^2 - 4\cancel{a}x - 6\cancel{a}\right] = \frac{1}{a} \times 0$

$x^2 - 4x - 6 = 0$

$x^2 - 4x - 6 = 0$

$\boxed{f(x) = x^2 - 4x - 6}$

19.

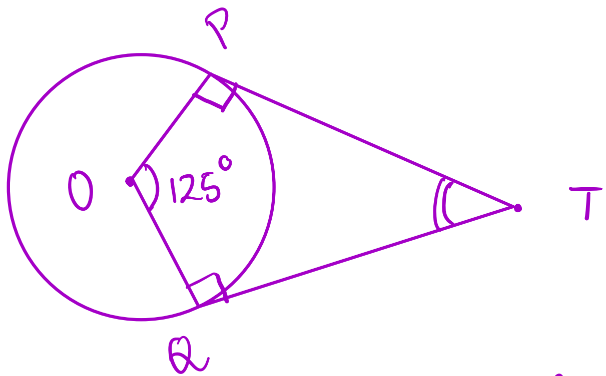


Circumference $= 2\pi r$

$= 2 \times \frac{22}{7} \times r = 132$

$r = \dfrac{\cancel{132}^{\cancel{66}\,3} \times 7}{\cancel{2} \times \cancel{22}} = \boxed{21\text{ cm}}$

20.



Angle sum property of a quadrilateral

$125^\circ + 90^\circ + \angle PTQ + 90^\circ = 360^\circ$

$\angle PTQ = 360^\circ - 180^\circ - 125^\circ$

$\boxed{\angle PTQ = 55^\circ}$

III.

21. $f(x) = x^2 - 7x - 4$, $\underbrace{\alpha^2+\beta^2}$

$f(x) = 0$,

$\rightarrow \underline{x^2 - 7x - 4 = 0}$ $\qquad ax^2+bx+c=0$

$$\alpha^2+\beta^2 = \underbrace{(\alpha+\beta)^2}_{\text{sum of roots}} - 2\cdot\underbrace{\alpha\cdot\beta}_{\text{product of roots}}$$

$-\frac{b}{a}$

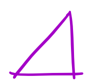
$\frac{c}{a}$

$$= (-7)^2 - 2\cdot(-4)$$

$$\boxed{\alpha^2+\beta^2 = 49+8 = 57}$$

22. $\cot\theta = \frac{7}{8}$, then find

$$\frac{(\underline{1+\sin\theta})(\underline{1-\sin\theta})}{(1+\cos\theta)(1-\cos\theta)}$$



$\sin\theta$
$\cos\theta$

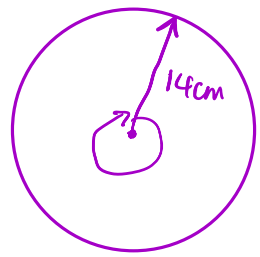
$\sin^2\theta + \cos^2\theta$

$$\frac{1^2-\sin^2\theta}{1^2-\cos^2\theta} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{(\cancel{\sin^2\theta}+\cos^2\theta) - \cancel{\sin^2\theta}}{(\sin^2\theta+\cancel{\cos^2\theta}) - \cancel{\cos^2\theta}}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$$

23.

$r = 14cm$



$\pi r^2 \rightarrow 60$ minutes

$\frac{\pi r^2}{60} \rightarrow$ 1 minute

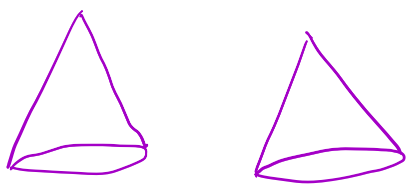
Req. area $= 5 \times \frac{\pi r^2}{60} = \cancel{5} \times \frac{22}{\cancel{7}} \times \cancel{14}^{2} \times \cancel{14}^{7} \times \frac{1}{\cancel{60}\ \cancel{30}\ \cancel{6}\ 3}$

$= \boxed{\frac{154}{3}\ cm^2}$

$51.\sim cm^2$

$10 \times \frac{\pi r^2}{60}$

24.



1 2

$\frac{r_1}{r_2} = \frac{2}{3}$

$\frac{h_1}{h_2} = \frac{3}{2}$

$\frac{V_1}{V_2} = \frac{\cancel{\frac{1}{3}} \cancel{\pi} r_1^2 h_1}{\cancel{\frac{1}{3}} \cancel{\pi} r_2^2 h_2}$

$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \left(\frac{2}{3}\right)^2 \times \frac{3}{2}$

$= \frac{2}{3} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2}}$

$\boxed{\frac{V_1}{V_2} = \frac{2}{3}}$

25.

Class	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
0-10	10	(0+10)/2 = 5	50
10-20	22	15	330
20-30	40	25	1000
30-40	16	35	560
40-50	12	45	540
	100		2480

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i}$$

$\cancel{\sum x_i}$

$$= \frac{2480}{100} = \boxed{24.8 = \bar{x}}$$

IV.

26. solve: $\frac{4}{x} + 3y = 14, \quad \frac{3}{x} - 4y = 23$

xy Let, $\frac{1}{x} = v$

$3 \times (4v + 3y = 14) \qquad (3v - 4y = 23) \times 4$

$12v + 9y = 42$

$12v - 16y = 92$

$\cancel{12v} + 9y = 42,$

$\cancel{12v} - 16y = 92$

(−) (+) (−)

$9y - (-16y) = 42 - 92$

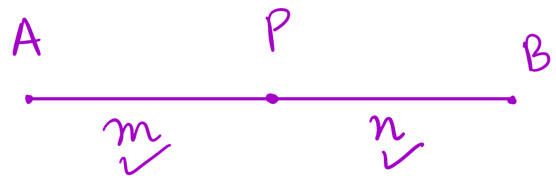
$$9y + 16y = -50$$
$$25y = -50$$
$$\boxed{y = -2}$$

$$4v + 3y = 14$$
Sub, $y = -2$
$$4v + 3(-2) = 14$$
$$4v - 6 = 14$$
$$4v = 14 + 6 = 20$$
$$4v = 20$$
$$v = \frac{20}{4} = 5$$

$$\boxed{v = 5}$$

$$v = \frac{1}{x} \quad , \quad \frac{1}{x} = 5$$
$$\therefore \boxed{x = \frac{1}{5}} , \boxed{y = -2}$$

27. $P(-4, 6)$, $A(-6, 10)$, $B(3, -8)$
(x, y) $\quad (x_1, y_1)$ $\quad (x_2, y_2)$



$m:n$

$\frac{m}{n}$

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n} , \frac{my_2 + ny_1}{m+n} \right)$$
$$\underbrace{(-4, 6)} = \left(\underbrace{\frac{m \times 3 + n \times (-6)}{m+n}} , \frac{m \times (-8) + n \times 10}{m+n} \right)$$

$$-4 = \frac{3m - 6n}{m+n}$$

$$3m - 6n = -4(m+n)$$

$$3m - 6n = -4m - 4n$$

$$3m + 4m = -4n + 6n$$

$$7m = 2n$$

$$\boxed{\frac{m}{n} = \frac{2}{7}} \qquad \boxed{m:n = 2:7}$$

28. $$\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

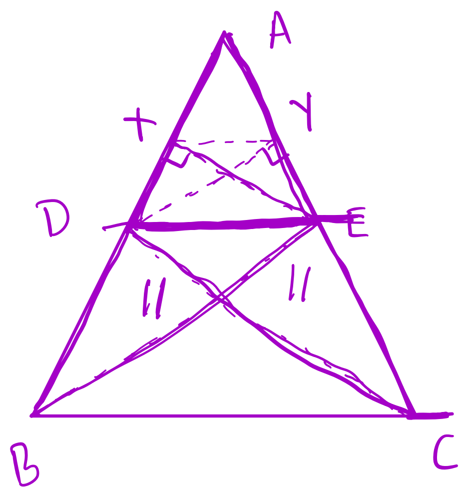
$$= \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{\cancel{2\sqrt{3}}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{\cancel{2\sqrt{3}}}}$$

$$= \frac{-4 + 3\sqrt{3}}{4 + 3\sqrt{3}} = \frac{(-4+3\sqrt{3})(-4+3\sqrt{3})(-1)}{(4+3\sqrt{3})(4-3\sqrt{3})}$$

$$= \frac{(-4+3\sqrt{3})^2(-1)}{4^2 - (3\sqrt{3})^2} = \frac{[(3\sqrt{3})^2 + 4^2 - 2 \times 4 \times 3\sqrt{3}][-1]}{16 - 27}$$

$$= \frac{(27+16-24\sqrt{3})}{\cancel{-}11} \times (\cancel{-}1) = \frac{(43-24\sqrt{3})}{11}$$

29.



To prove —

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\triangle BDE$, $\triangle BCE$, $\triangle ADE$

Construction — Draw $EX \perp AD$, $DY \perp AE$

$$\frac{ar(\triangle ADE)}{\boxed{ar(\triangle BDE)}} = \frac{\frac{1}{2} \times AD \times EX}{\frac{1}{2} \times BD \times EX} \quad \text{— (i)}$$

$$\frac{ar(\triangle ADE)}{\boxed{ar(\triangle CDE)}} = \frac{\frac{1}{2} \times AE \times DY}{\frac{1}{2} \times CE \times DY} \quad \text{— (ii)}$$

$ar(\triangle BDE) = ar(\triangle CDE)$ [triangles on the same base and between two parallel lines have equal area]

$$\boxed{\frac{AD}{BD} = \frac{AE}{CE}}$$

Hence, proved!

30. $9m$, $9m+1$, or $9m+8$.

what's Euclid's division lemma

$x = y \cdot q + r$, $\quad 0 \le r < y$.

$x^3 \longrightarrow 9m$ or $9m+1$, $9m+8$.

$2^3 = 8 = 2\cdot 3 + 2 = 3 \cdot 2 + 2$ q

$3^3 = 27 = 3 \cdot 9 + 0$ q

$4^3 = 64 = 3 \cdot 21 + 1$ q q r

Let,

$\underbrace{x = 3 \cdot q + r}$ y $\quad 0 \le r < 3$

x^3 as ____

$r = 0$

$r = 1$

$r = 2$

Case - I $\quad r = 0$

$x = 3 \cdot q + 0 = 3q$

Cubing both sides. $\quad x^3 = (3q)^3$

$x^3 = 3^3 q^3$

$= 9 \cdot (3q^3)$

$m = 3q^3 \quad \boxed{x^3 = 9 \cdot m}$ — (i)

Case - II $\quad r=1$

$$x = 3\cdot q + 1$$

Cubing both sides

$$x^3 = (3q+1)^3 = (3q)^3 + 1^3 + 3\cdot 3q\cdot 1\,(3q+1)$$

$$x^3 = 3^3q^3 + 1 + 9q(3q+1)$$

$$x^3 = \underbrace{27q^3 + 1 + 27q^2 + 9q}$$

$$x^3 = 9q(3q^2+3q+1) + 1$$

$$x^3 = \underline{9(3q^3+3q^2+q)}+1$$

Let, $m = 3q^3+3q^2+q$,

$$x^3 = 9(m)+1$$

$$\boxed{x^3 = 9m+1} \quad \text{— (ii)}$$

Case - III $\quad r=2$.

$$x = 3\cdot q + 2$$

$$x^3 = (3q+2)^3 = 3^3q^3 + 2^3 + 3\cdot 3q\cdot 2\,(3q+2)$$

$$= \underbrace{27q^3 + 8 + 54q^2 + 36q}$$

$$= \underbrace{9(3q^3+6q^2+4q) + 8}$$

$m = 3q^3+6q^2+4q$, $\quad \boxed{x^3 = 9m+8} \quad$ — (iii)

31. Solve: $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$. $(x \neq -4, 7)$

$$\frac{1(x-7) - 1(x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{\cancel{x} - 7 - \cancel{x} - 4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-\cancel{11}}{(x+4)(x-7)} = \frac{\cancel{11}}{30}$$

$$\underbrace{(x+4)(x-7)} = -30$$

$$x^2 + 4x - 7x - 28 = -30$$

$$x^2 - 3x - 28 + 30 = 0$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - x - 2x + 2 = 0$$

$$x(x-1) - 2(x-1) = 0$$

$$(x-1)(x-2) = 0$$

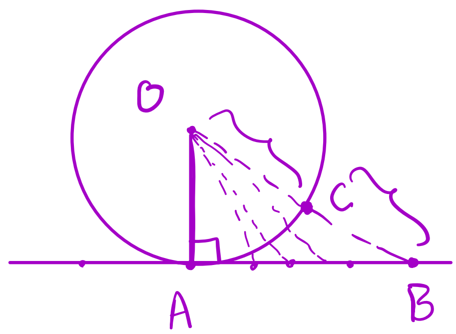
$x - 1 = 0$ or $x - 2 = 0$

$x = 1,$ $x = 2$

$\boxed{x = 1, 2}$

Ans.

32.



To prove

$OA \perp AB$

$\boxed{OA = OC}$ [Radius]

$OB = OC + CB$ $\boxed{CB > 0}$

$OB > OC$

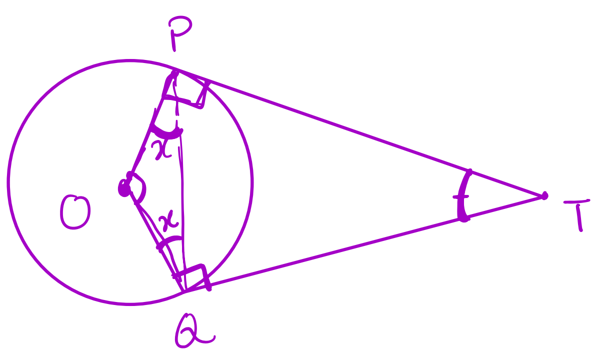
$OB > OA$

OA is the shortest line segment.

But, the shortest distance of a point from a line is the perpendicular distance from that line.

$OA \perp AB$. Hence, proved.

OR



To prove

$\angle PTQ = 2\angle OPQ$

ΔOPQ, $OP = OQ$ [Radius]

$\angle OPQ = \angle OQP$ [Isosceles triangle]

In quadrilateral $OPTQ$,

$\angle PTQ + 90^\circ + \angle POQ + 90^\circ = 360^\circ$

$\angle PTQ + \angle POQ = 360^\circ - 180^\circ = 180^\circ$ — (i)

ΔPOQ,

$\angle POQ + x + x = 180^\circ$ [Angle sum of a triangle]

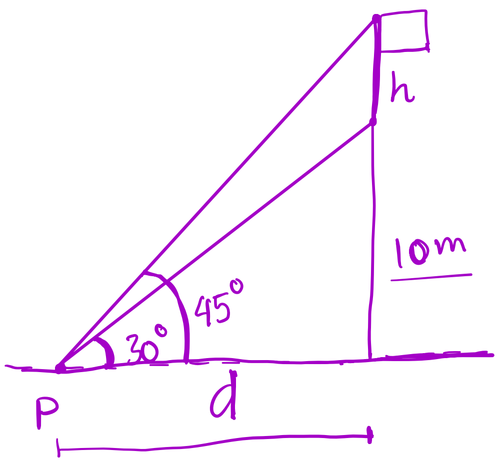
$\angle POQ + 2x = 180^\circ$ — (ii)

$\angle PTQ + \cancel{\angle POQ} = \cancel{\angle POQ} + 2x$

$\angle PTQ = 2x$, $x = \angle OPQ$

$\therefore \angle PTQ = 2\angle OPQ$. Hence, proved.

33.



$\tan 30^\circ = \frac{10}{d}$

$d = \frac{10}{\tan 30^\circ} = \frac{10}{1/\sqrt{3}}$

$= 10\sqrt{3}$ m

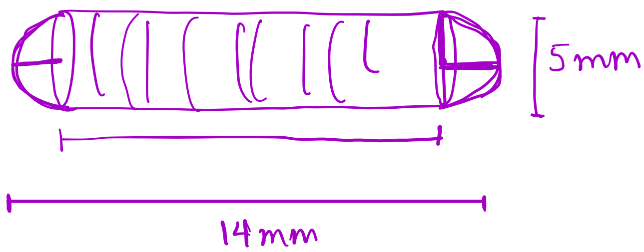
$= 17.32$ m

$$\tan 45^\circ = \frac{10+h}{d}$$

$$d \cdot \tan 45^\circ = 10+h,$$

$$h = d-10$$
$$= 17.32-10$$
$$h = 7.32 \text{ m}$$

34.



$$r = \frac{5\text{mm}}{2}$$
$$= 2.5 \text{ mm}$$

Surface area = lateral surface area of the cylinder + curved surface area of the two hemispheres

$$= 2\pi r h + 4\pi r^2$$

$$h = 14 - 2\times 2.5 = 14-5$$
$$= 9 \text{ mm}$$

$$SA = 2\times\pi\times 2.5\times 9 + 4\times\pi\times(2.5)^2$$
$$= 70\pi \text{ mm}^2$$
$$= \overset{10}{\cancel{70}} \times \frac{22}{\cancel{7}} = 220 \text{ mm}^2$$

Volume = volume of the cylinder + volume of two hemispheres

$$= \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi \times (2.5)^2 \times 9 + \frac{4}{3} \times \pi \times (2.5)^3 \quad mm^3$$

$$= \frac{925}{12}\pi \; mm^3 .$$

35\. (i) A king of red color

4 kings. 2 red. 2 black

$$\frac{2}{52} = \frac{1}{26}$$

(ii) The Jack of hearts

$$\frac{1}{52}$$

(iii) An ace,

$$\frac{4}{52} = \frac{1}{13}$$

(iv) A face card

$$\frac{12}{52} = \frac{3}{13} .$$

36.

$n = 30$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Class	Frequency	Cumulative f
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30

median class

$\frac{n}{2} = 15$

l = lower limit of the median class
$= 55$

cf = cumulative f of class preceding median class
$= 13$

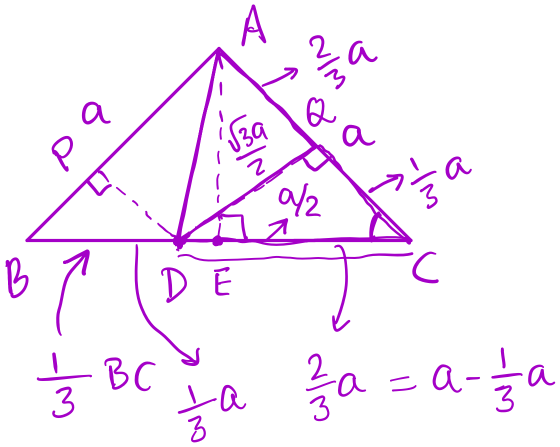
f = frequency of median class
$= 6$

h = class size, height
$= 60 - 55 = 5.$

median
$$= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$
$$= 55 + \frac{(15-13)}{6} \times 5$$
$$= 55 + \frac{2}{6} \times 5$$
$$= \frac{170}{3} \approx \boxed{56.67 \text{ kg}}$$

VI.

37.



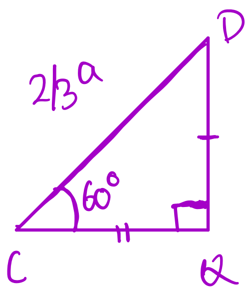
Let side of $\Delta ABC = a$

To prove —

$9AD^2 = 7AB^2$

$$h = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \sqrt{a^2 - \frac{a^2}{4}} = \sqrt{\frac{4a^2 - a^2}{4}}$$

$$= \sqrt{\frac{3a^2}{4}} = \frac{\sqrt{3}\,a}{2}$$



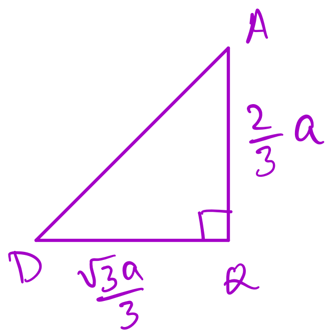
$$\sin 60° = \frac{DQ}{\frac{2}{3}a}$$

$$\frac{\sqrt{3}}{\cancel{2}} = \frac{DQ \times 3}{\cancel{2}a}$$

$$DQ = \frac{\sqrt{3}a}{3}$$

$$\cos 60° = \frac{CQ}{\frac{2}{3}a}$$

$$\frac{1}{\cancel{2}} = \frac{CQ \times 3}{\cancel{2}a}, \quad CQ = \frac{1}{3}a$$



$$AD^2 = AQ^2 + DQ^2$$

$$AD^2 = \left(\frac{2}{3}a\right)^2 + \left(\frac{\sqrt{3}a}{3}\right)^2$$

$$AD^2 = \frac{4a^2}{9} + \frac{3a^2}{9}$$

$$AD^2 = \frac{7a^2}{9}$$

$\therefore \quad 9AD^2 = 7a^2, \qquad a = AB = BC = AC$

$$\boxed{9AD^2 = 7AB^2}$$

Hence, proved.

38.

$a_n = 3+2n$

$a_{24} = 3+2\times 24 = 3+48 = \boxed{51}$

$S_{48} = ?$

$a_1 = 3+2\times 1$
$= 5$

$a_{48} = 3+2\times 48$
$= 3+96$
$= 99$

$S_n = \frac{n}{2}(a_1+a_n)$

$S_{48} = \frac{48}{2}(a_1+a_{48})$

$= \frac{48}{2}(5+99)$

$= 24\times 104 = \boxed{2496}$

Let, the number be a

$a - \frac{1}{a} = 58\frac{58}{59}$

$\frac{a^2-1}{a} = \frac{3480}{59} = \frac{(58\times 59+58)}{59}$

$\frac{a^2-1}{a} = \frac{3480}{59}$

$59a^2 - 59 = 3480a$

$59a^2 - 3480a - 59 = 0$

$\frac{-b \pm \sqrt{a^2-4ac}}{2a}$

$$\frac{-(-3480) \pm \sqrt{59^2 - 4 \times 59 \times (-59)}}{2 \times 59}$$

$$\frac{3480 \pm \sqrt{12124324}}{2 \times 59} = \frac{3480 \pm 3482}{118}$$

$$\frac{6962}{118} = 59 \qquad -\frac{1}{59}$$

$$a_1 \qquad a_2$$

$$59 - \frac{1}{59} = 0 \qquad -\frac{1}{59} - (-59)$$

$$\boxed{59 - \frac{1}{59}}$$